Technique for Hessian Matrix Formation in Transverse Gradient Coil Optimization

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Abstract: Target field approaches are powerful for gradient coil design. However, these methods usually need predefined coil geometries, which is not always possible. Mathematical optimization methods may be advantageous in these cases. Because wire segments move at different rates in multi-dimensional vector space, segments may cross each other and result in impractical design. Here we demonstrate a method for conjugate gradient descent that can help mitigate this problem. The method may be helpful in other mathematical optimization methods for gradient coil design.

Theory and Method

A general description of conjugate gradient descent is given in Ref. 3, which constructs two sequences of vectors using Eqs. 1-3

$$\boldsymbol{g}_{i+1} = \boldsymbol{g}_i - \lambda_i \boldsymbol{A}\, \boldsymbol{d}_i \qquad [1]$$

$$\boldsymbol{d}_{i+1} = \boldsymbol{g}_{i+1} + \gamma_i \boldsymbol{d}_i \qquad [2]$$

$$\gamma_i = (\boldsymbol{g}_{i+1} - \boldsymbol{g}_i)\, \boldsymbol{g}_{i+1} / (\boldsymbol{g}_i\ \boldsymbol{g}_i) \qquad [3]$$

where $\boldsymbol{g}_i$ is the gradient vector, $\boldsymbol{d}_i$ is a vector describing the search directions in vector space, and $\boldsymbol{g}_i\, \boldsymbol{g}_j = 0$, $\boldsymbol{d}_i\, \boldsymbol{A}\ \boldsymbol{d}_j = 0$, $\boldsymbol{g}_i\, \boldsymbol{d}_j = 0$. In gradient coil optimization, gradient vector $\boldsymbol{g}$ can be derived based on the defined cost function and Biot-Savart's law. Practical designs can be obtained without knowing the Hessian matrix, but in our view the construction of $\boldsymbol{A}$ is a serious concern in gradient coil design. For cylindrical z gradient coil design, γ_i is one-dimensional, but for transverse coil design, it is two-dimensional (φ,z) or three-dimensional (r,φ,z). Correspondingly, each wire segment moves in two or three-dimensional vector space. Wire segment coordinates $\boldsymbol{P}_i(r,\varphi,z)$ can be searched iteratively based on (1,2)

$$\boldsymbol{P}_{i+1}(r,\varphi,z) = \boldsymbol{P}_i(r,\varphi,z) + \underline{\alpha}\ \boldsymbol{d}_i(r,\varphi,z) \qquad [4]$$

where $\underline{\alpha}$ is a matrix describing searching steps along r, φ and z during coil optimization. At each iteration, the choice of matrix $\underline{\alpha}$ is critical to reach a local minima constrained by edge conditions. This is equivalent to construction of the Hessian matrix $\boldsymbol{A}$ in Eq. 1 for the next iteration. Neglect of the Hessian matrix can result in a design as seen in Fig.1. The sharp curvatures arise from local minima for specific values of φ and z as determined by the geometry and Biot-Savart based error function. They do not depend on the number of wire segments. This can be avoided by constructing matrix $\underline{\alpha}$ based on the g_φ/g_z ratio. In practice, the gradient vectors $\boldsymbol{g}_{\varphi i}$ are 100 to 1000 times larger than $\boldsymbol{g}_{zi}$, depending on the optimization stages. By using this technique, it is possible to generate a practical design without loss of the conjugacy property and make more efficient gradient coils.

Results and Discussion

A 12.5cm inner diameter three axial torque-balanced cylindrical gradient coil based on this technique has been successfully designed, evaluated and routinely applied in MRI experiments with rats in our lab. Figures 1 and 2 show the winding patterns for transverse gradients with and without using this technique. For Fig. 2, the efficiency is 15.2 gauss/cm at 100A. The irregular winding pattern is apparent, and the efficiency is difficult to increase. For Fig. 2, the efficiency is 21.3 gauss/cm at 100A, and the winding patterns are smoother. The measured rise time was less than 96 μs in all three axes. The measured rms error over a volume of 7.0 cm diameter sphere was 1.9%. The technique described above has also been applied where r is a parameter in the design of a short efficiency-optimized torque-balanced local gradient coil for the human brain.

Discussion

In transverse gradient coil optimization using conjugate gradient descent, the search steps in each iteration are critical and require determination of the Hessian matrix $\boldsymbol{A}$ in Eq. 1 for the next iteration. It is expected that the technique described here can be applied to planar gradient coil design and may be useful for other optimization techniques.

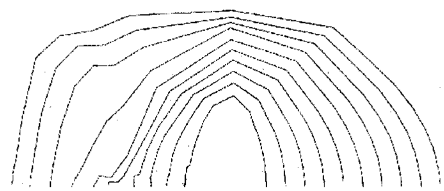

Figure 1. Winding pattern without using the proposed technique. Efficiency is 15.2 guass/cm/100A. Sharp curvatures can be appreciated.

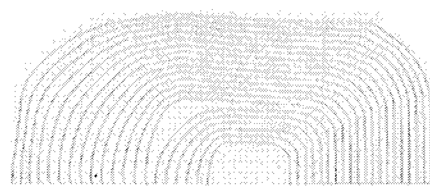

Figure 2. Winding pattern based on the proposed technique.
The efficiency is 21.3 guass/cm/100A; the measured rise time is less than 96μs.

References

1. Wong, E.C., Jesmanowicz, A., Hyde, J.S., *Magn. Reson. Med.*, 21, 39-48, 1991.
2. Brey, W.W., Mareci, T.H., Daughty, J., *J. Magn. Reson.*, B112, 124-130, 1996.
3. Press, W.H. *et al.*, *Numerical Recipes in C*, 2nd edition. Cambridge University press, 1992.